

# Intermittency in a single event <sup>\*</sup>

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February 1996

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## Abstract

The possibility to study intermittency in a single event of high multiplicity is investigated in the framework of the  $\alpha$ -model. It is found that, for cascade long enough, the dispersion of intermittency exponents obtained from individual events is fairly small. This fact opens the possibility to study the distribution of the intermittency parameters characterizing the cascades seen (by observing intermittency) in particle spectra.

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1. The original suggestion of intermittent behaviour in multiparticle production at high energies [1] was based on analysis of a single event of very high multiplicity recorded by the JACEE collaboration [2]. It was soon realized, however, that the idea can be applied to events of any multiplicity provided that a proper averaging of the distributions is performed [3]. This led to many successful experimental studies of intermittency [4], and allowed to express the effect in terms of the multiparticle correlation functions[5]. It should be realized, however, that the averaging procedure, apart from clear advantages, brings also a danger of overlooking some interesting effects if they are present only in a part of events produced in high-energy collisions. It seems therefore interesting to study intermittency parameters of individual events, hoping that they may indicate some specific production mechanism (a typical example is the production of quark-gluon plasma which is expected to be characterized by specific intermittency exponents, see e.g. [6], and certainly not expected to be present in each event).

Such studies should necessarily be restricted to high-multiplicity events because only there one may expect the statistical fluctuations to be under control. However, even neglecting statistical errors due to the finite number of particles, there remains an intrinsic uncertainty of the intermittency parameters: the cascade responsible for intermittent behaviour has different realizations in different events. As the intermittency exponents determined from different realizations of the same random cascade are expected to scatter around the average, the method has a finite resolution with respect to the parameters of the random cascade. Clearly, the resolution is a function of the number of steps in the cascade.

In the present paper we investigate the distribution of intermittency exponents obtained from analysis of individual events, using as a tool the  $\alpha$ -model of one-dimensional random cascade [1]. We concentrate on two problems :

(a) how much the average value of an intermittency exponent obtained from analysis of individual events differs from its "theoretical" value cal-

culated from the assumed parameters of the random cascade and from the "standard" value obtained by averaging factorial moments over many events.

(b) what is the dispersion of this distribution or, in other words, what is the resolution of the measurement and how it depends on the number of steps in the cascade.

We have investigated cascades of up to 10 steps. Our results are described in some detail below. They can be summarized as follows :

(i) the average value of the intermittency exponent reproduces well the value obtained by averaging the factorial moments over events. This result weakly depends on the number of steps in the cascade. However, both numbers have a tendency to underestimate the theoretical value for cascades of short lengths.

(ii) the dispersion of the distribution is inversely proportional to the length of the cascade. For cascades of less than 6 steps it becomes fairly large, but still substantially smaller than the average value.

We thus find that a measurement of the distribution of intermittency exponents obtained from individual events of high multiplicity may indeed help to reveal existence of groups of events emerging from cascades with different characteristics.

**2.** The  $\alpha$ -model of random cascading [1] describes a multiparticle event as a series of steps in which each phase-space interval is divided into some number of equal parts. At any step  $n$  ( $n = 1, 2, \dots, N$ ) particle density in each of the parts is obtained by multiplication of the density at the  $(n-1)$ th step by one of the two values  $(a, b)$  of random variable  $W$  with the probabilities  $\alpha$  and  $\beta$ , respectively. For simplicity one assumes also :

$$\langle W \rangle = \alpha a + \beta b = 1 \quad (1)$$

where  $\langle \rangle$  denotes the average value henceforth. Note that (1) implies :

$$\alpha = \frac{b-1}{b-a}, \beta = \frac{1-a}{b-a} \quad (2)$$

So that the model is defined by two parameters  $a$  and  $b$ .

In our simulation of the  $\alpha$ -model we have divided each bin into 2 parts, so the number of bins at each step equals :

$$M(n) = 2^n \quad (3)$$

and thus the length of each bin is equal to :

$$d(n) = \frac{D}{M(n)} \quad (4)$$

where  $D$  is the total phase-space interval.

The "standard" method is to study scaling behaviour of the normalized moments of particle densities :

$$\langle Z_m^q(d) \rangle = \langle (x_m(d))^q \rangle \quad (5)$$

Here  $x_m(d)$  is the density obtained after  $n$  steps of the cascade in the  $m$ th bin ( $m = 1, \dots, M(n)$ ), and the average is taken over all considered events. It follows from (1) that  $\langle Z_m^1 \rangle = 1$ . In the  $\alpha$ -model  $\langle Z_m^q(d) \rangle$  follows the power law :

$$\langle Z_m^q(d) \rangle = \left(\frac{2D}{d}\right)^{\varphi_q} \quad (6)$$

where the intermittency exponents are given by :

$$\varphi_q = \log_2 \langle W^q \rangle \quad (7)$$

**3.** If one is interested in event-by-event analysis, one is forced to consider the so-called horizontal average  $Z^q(d)$  [1] :

$$Z^q(d) = M^{-1} \sum_{m=1}^M Z_m^q(d) \quad (8)$$

obtained by averaging over all bins. In the  $\alpha$ -model the average particle density is independent of  $m$  and thus  $Z^q(d)$  follows the same scaling law (6) as  $Z_m^q(d)$  :

$$Z^q(d) = \left(\frac{2D}{d}\right)^{\varphi_q} \quad (9)$$

As we have already explained,  $\varphi_q$  calculated from (9) fluctuate from event to event even for fixed parameters of the cascade. Its average over many events should approach the value given by (7). The dispersion around the average, however, does not vanish, even in the limit of infinite number of events. In other words, even for events with very large multiplicity we cannot determine intermittency exponents with arbitrarily high precision: there is a "natural" uncertainty of this measurement. This uncertainty is expected to decrease with increasing number of steps in the cascade. Furthermore, the dispersion of the distribution of the factorial moment  $Z_q(d)$  can be estimated as:

$$D^2(Z^q(d)) \simeq \text{const} \quad (10)$$

which explicitly shows that  $D(\varphi_q)$  is inversely proportional to the length of the cascade.

We have performed numerical simulations of the  $\alpha$ -model in order to obtain the feeling to what extent these theoretical prejudices are realized in practice. In Figs. 1, 2 the histograms of the values of intermittency exponent  $\varphi_2, \varphi_3$  are plotted for 5000 generated cascades with 6 and 10 steps. These numbers were determined for each event using the method described in [1] (and applied there to the JACEE event [2]). One sees that both the average value and the dispersion depend on number of steps in the cascade. For small number of steps the average value obtained from simulation is smaller than the "true" value given by Eq. (7). For 10 steps, however, the simulation gives the average rather close to the theoretical result. The dispersion of the distribution estimated directly from the observed peak, decreases with the number of cascade steps following well the  $1/n$  rule of the Eq.(10). Its numerical value as a function of the cascade length is presented in Tables 1, 2 for 2 different sets of cascade parameters a, b. The dispersion is relatively small, and it allows to distinguish between the cascades with different parameters (Figs.1, 2).

Table 1. Intermittency exponents and their dispersions for  $a = 0.8$ ,  $b = 1.1$  and  $n = 5, \dots, 10$  cascade steps

	theor.	5	6	7	8	9	10
$\varphi_2 = 10^{-2} \times$	2.9	$2.4 \pm 0.9$	$2.5 \pm 0.8$	$2.6 \pm 0.7$	$2.7 \pm 0.6$	$2.7 \pm 0.6$	$2.7 \pm 0.5$
$\varphi_3 = 10^{-2} \times$	8.2	$6.9 \pm 2.6$	$7.1 \pm 2.3$	$6.6 \pm 2.0$	$7.7 \pm 1.7$	$7.7 \pm 1.6$	$7.8 \pm 1.5$

Table 2. Intermittency exponents and their dispersions for  $a = 0.5$ ,  $b = 1.5$  and  $n = 5, \dots, 10$  cascade steps

	theor.	5	6	7	8	9	10
$\varphi_2 = 10^{-1} \times$	3.2	$2.4 \pm 1.0$	$2.5 \pm 1.0$	$2.5 \pm 0.8$	$2.7 \pm 0.7$	$2.7 \pm 0.6$	$2.8 \pm 0.6$
$\varphi_3 = 10^{-1} \times$	8.1	$5.9 \pm 2.3$	$6.1 \pm 2.2$	$6.4 \pm 2.1$	$6.7 \pm 1.8$	$6.7 \pm 1.6$	$6.8 \pm 1.6$

4. Our conclusions can be summarized as follows :

(a) the average value of the intermittency exponent obtained from our analysis is fairly close to the "theoretical" value.

(b) the dispersion of the distribution is inversely proportional to the length of the cascade. It is found to be relatively small. This allows to distinguish between cascades with reasonably different parameters.

### Acknowledgements

The research was supported in part by the KBN grant No. 2 PO 3B 08308 and by the European Human Capital and Mobility Program ERBCIPDCT940613.

## References

- [1] A. Bialas, R. Peschanski, *Nucl. Phys. B* **273**, 703 (1986)

- [2] JACEE coll. T. H. Burnett et al. *Phys. Rev. Letters* 50 (1983) 2062
- [3] B. Buschbeck, P. Lipa, *Phys. Lett. B*223 (1989) 465 and private communication
- [4] for recent reviews see  
     E. A. de Wolf, I. M. Dremin, W. Kittel, *HEN-362 (1993) update July 1995*,  
     P. Bozek, M. Ploszajczak, R. Botet, *Phys. Rep.* 252 (1995) 101-176
- [5] P. Carruthers, I. Sarcevic, *Phys. Lett. B*189 (1987) 442
- [6] A. Bialas, R. Hwa, *Phys. Lett. B*253 (1991) 436

### Figure captions

**Fig. 1** Distribution of the intermittency exponent  $\varphi_2$  as determined in individual events generated from the  $\alpha$ -model. 5000 events with  $a = 0.8, b = 1.1$  (a) and 5000 events with  $a = 0.5, b = 1.5$  (b) were used. Cases (a) and (b) are plotted in two different scales. Histogram for the case (a) is multiplied by  $10^{-1}$ .

Solid line and dots : 10 cascade steps, dashed line and crosses : 6 cascade steps.

**Fig. 2** Distribution of the intermittency exponent  $\varphi_3$ . Other details as in Fig. 1.